Caltech

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Understanding Quantum Gravity

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Black Holes

Black Holes are hard to understand!

- Massive – strong gravity
- Black hole evaporation – quantum mechanical

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General Relativity + Quantum Field Theory

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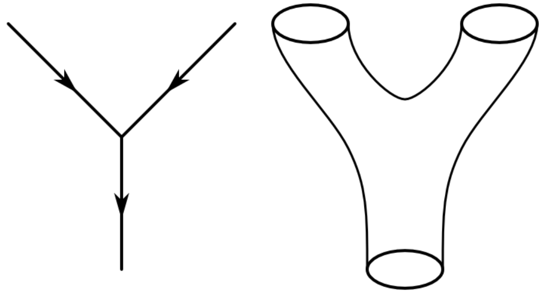
Quantum Gravity!

Quantum Gravity



- **Quantum Field Theory**
  - Quantizes a point particle – the study of their worldline
  - UV divergence
- **String theory**
  - A 10d theory that is constructed via quantization of strings (1d object) – the dynamics is captured by the 2d world sheet traced out by the string moving through spacetime
  - UV-complete without spacetime divergence
  - The particles that we observe = the Fourier modes of string quanta at a large distance.
  - Looks like a point particle (Cannot resolve the extent of the string at such a low scale.)
  - Folklore theorem of quantum gravity: the spectrum is comprised of massless states and their towers of excitations thereby giving a complete set of the spectrum of particles

Gravity and gauge theory in Strings?

Closed strings → The massless modes corresponding to a “quantized” graviton as a symmetric two-tensor

Open strings → The gauge bosons of Yang–Mills theory

F-theory

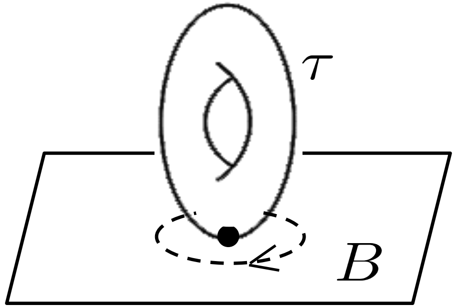
- F-theory: a mathematical toolbox to study string theory (type IIB)
- Fundamental tools to describe the basic interactions in physics
  - Gauge theory
  - Representation theory
- F-theory enables to have geometric perspective for analyzing gauge theory and representation theory of its low energy EFT.
- Elliptic Fibrations are used for geometric engineering of physical theories.

F-theory Construction

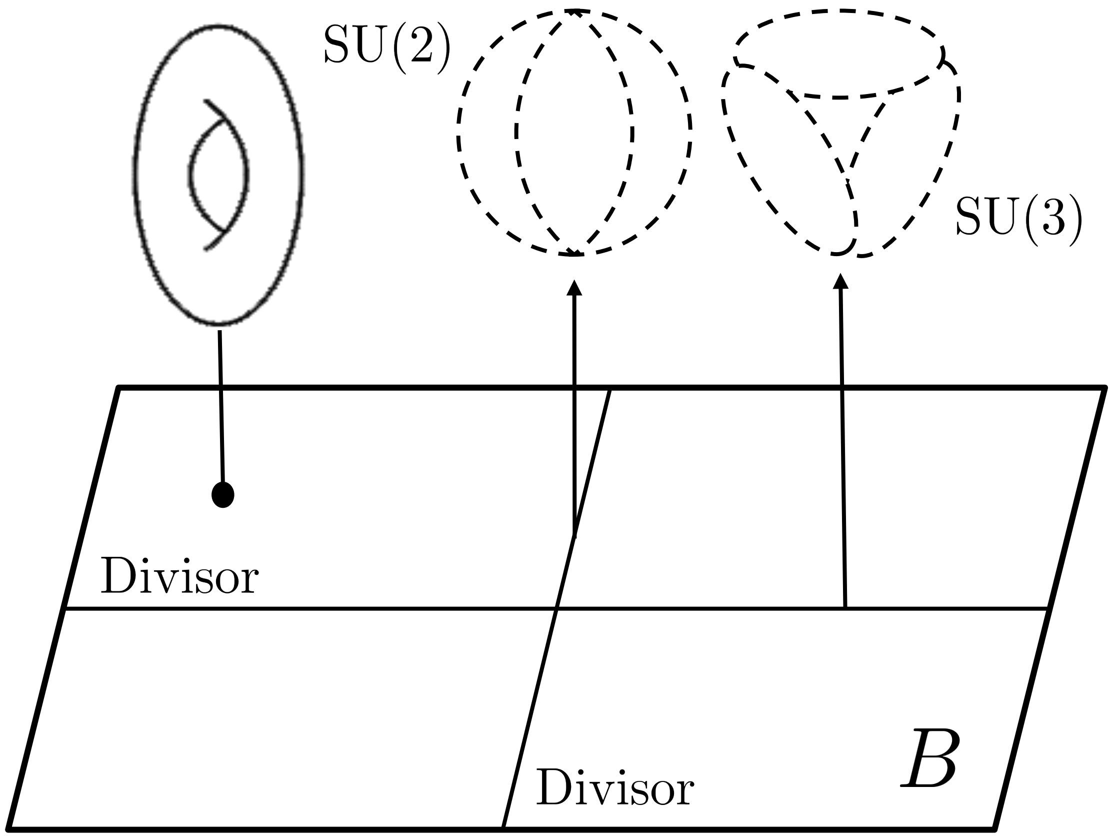
0	1	2	3	4	5	6	7	8	9	10	11
$\mathbb{R}^4$				Base ($B_3 \supset D_i$)						Elliptic Fiber	
4d spacetime				internal dimensions of 7-brane				Transverse dimensions orthogonal to 7-brane		Axio-dilaton	

- Strings compactifications:

$$\text{F-theory} \xrightarrow{CY_3} 6d \text{ EFT} \qquad \text{F-theory} \xrightarrow{CY_4} 4d \text{ EFT}$$



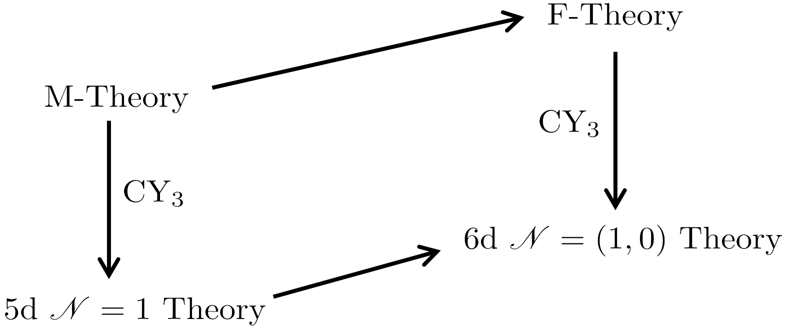
Gauge theories via elliptic fibration



Elliptic Fibration	Gauge Theory
Codim 1 singularities	Gauge algebra ($\mathfrak{g}$)
Codim 2 singularities	Representation ($\mathbf{R}$)
Crepant resolutions	Coulomb phases
Flops	Phase transitions
Triple intersection	5d prepotential
Mordell-Weil group	The fundamental group of the gauge group ($\pi_1(G)$)

5d/6d Supergravity

- Elliptic Fibrations
  - Geometrically engineered gauge theories
  - Captures global aspect of the gauge theory



F-theory on Y $\downarrow$ 6d $\mathcal{N} = (1,0)$ sugra	M-theory on Y $\downarrow$ 5d $\mathcal{N} = 1$ sugra	F-theory on $Y \times S^1$ $\downarrow$ 5d $\mathcal{N} = 1$ sugra
$n_V^{(6)} = h^{1,1}(Y) - h^{1,1}(B) - 1$ $n_H^0 = h^{2,1}(Y) + 1$ $n_T = h^{1,1}(B) - 1$	$n_V^{(5)} = n_V^{(6)} + n_T + 1 = h^{1,1}(Y) - 1$ $n_H^0 = h^{2,1}(Y) + 1$	

$n_H = n_H^0 + n_H^{ch}$ $\quad$ (n_H^0 : neutral hypermultiplet, n_H^{ch} : charged hypermultiplet)

- 5d/6d Field Contents

5d $\mathcal{N} = 1$ Theory	6d $\mathcal{N} = (1,0)$ Theory
Gravity multiplets $(g_{\mu\nu}, \psi_{\mu I}, A_\mu)$	Gravity multiplets $(g_{\mu\nu}, B^+_{\mu\nu}, \psi^-_\mu)$
	Tensor multiplets $(B^-_{\mu\nu}, \phi, \chi^+)$
Vector multiplets $(A^A_\mu, \lambda^A_I, \phi^A)$	Vector multiplets (A_μ, λ^-)
Hypermultiplets (ζ^m, A^m_I)	Hypermultiplets (q, η^+)

Vector multiplets $\rightarrow$ Weyl Chamber
Massless hypermultiplets at the singularities $\rightarrow$ subchamber structures

Necessary geometric data

- Required data to determine the spectra:
  - Euler characteristic χ and Hodge numbers $h^{1,1},\ h^{2,1}$
  - Triple intersection polynomial
- How do we compute these?
  - Compute in the resolved space and then pushforward to the base.

Pushforward formula-1

- The following theorem gives the total Chern class after a blowup along a local complete intersection.

Theorem (Aluffi). *Let $Z \subset X$ be the complete intersection of d nonsingular hypersurfaces $Z_1, \ldots, Z_d$ meeting transversally in X. Let $f : \widetilde{X} \longrightarrow X$ be the blowup of X centered at Z. We denote the exceptional divisor of f by E. The total Chern class of $\widetilde{X}$ is then:*

$$c(T\widetilde{X}) = (1+E)\left(\prod_{i=1}^{d} \frac{1+f^*Z_i - E}{1+f^*Z_i}\right) f^*c(TX).$$

Pushforward formula-2

- The following theorem provides a user-friendly method to compute invariants of the blown-up space in terms of the original space.

Theorem (Esole–Jefferson–Kang). *Let the nonsingular variety $Z \subset X$ be a complete intersection of d nonsingular hypersurfaces $Z_1, \ldots, Z_d$ meeting transversally in X. Let E be the class of the exceptional divisor of the blowup $f : \widetilde{X} \longrightarrow X$ centered at Z. Let $\widetilde{Q}(t) = \sum_a f^* Q_a t^a$ be a formal power series with $Q_a \in A_*(X)$. We define the associated formal power series $Q(t) = \sum_a Q_a t^a$, whose coefficients pullback to the coefficients of $\widetilde{Q}(t)$. Then the pushforward $f_* \widetilde{Q}(E)$ is*

$$f_* \widetilde{Q}(E) = \sum_{\ell=1}^{d} Q(Z_\ell) M_\ell, \quad \text{where} \quad M_\ell = \prod_{\substack{m=1 \\ m \neq \ell}}^{d} \frac{Z_m}{Z_m - Z_\ell}.$$

Pushforward formula-3

- This theorem gives a simple method to pushforward analytic expressions in the Chow ring of the projective bundle X_0 to the Chow ring of its base.
- It is a direct consequence of functorial properties of the Segre class.

Theorem (Esole–Jefferson–Kang). *Let $\mathscr{L}$ be a line bundle over a variety B and $\pi : X_0 = \mathbb{P}[\mathscr{O}_B \oplus \mathscr{L}^{\otimes 2} \oplus \mathscr{L}^{\otimes 3}] \longrightarrow B$ a projective bundle over B. Let $\widetilde{Q}(t) = \sum_a \pi^* Q_a t^a$ be a formal power series in t such that $Q_a \in A_*(B)$. Define the auxiliary power series $Q(t) = \sum_a Q_a t^a$. Then*

$$\pi_* \widetilde{Q}(H) = -2 \left. \frac{Q(H)}{H^2} \right|_{H=-2L} + 3 \left. \frac{Q(H)}{H^2} \right|_{H=-3L} + \frac{Q(0)}{6L^2},$$

where $L = c_1(\mathscr{L})$ and $H = c_1(\mathscr{O}_{X_0}(1))$ is the first Chern class of the dual of the tautological line bundle of $\pi : X_0 = \mathbb{P}(\mathscr{O}_B \oplus \mathscr{L}^{\otimes 2} \oplus \mathscr{L}^{\otimes 3}) \to B$.

How can this be used?

- Topological invariants for threefolds 1703.00905 (Esole, Jefferson, MJK)
- Various characteristic numbers for fourfolds 1807.08755, 1808.07054 (Esole, MJK)
- 5d/6d spectra and the geometry of Coulomb branch
  - Simple models: G_2 1805.03214 (Esole, Jagadeesan, MJK), F_4 1704.08251 (Esole, Jefferson, MJK)
  - Semi-simple models: SO(4) and Spin(4) 1802.04802 (Esole, MJK),

    SU(2)xG_2 1805.03214 (Esole, MJK), SU(2)xSU(3) 1905.05174 (Esole, Jagadeesan, MJK),

    SU(2)xSp(4), (SU(2)xSp(4))/Z_2, SU(2)xSU(4), (SU(2)xSU(4))/Z_2 1712.02337 (Esole, MJK, Yau)
- Non-trivial Mordell-Weil group: U(1) 1410.0003 (Esole, MJK, Yau),

  Torsions (Z_2, Z_3) 1802.04802, 1808.07054 (Esole, MJK), 1712.02337 (Esole, MJK, Yau).

Simple Groups

➢ Euler characteristics and Hodge numbers of Calabi-Yau threefolds are computed for various simple groups. [Esole, Jefferson, MJK]

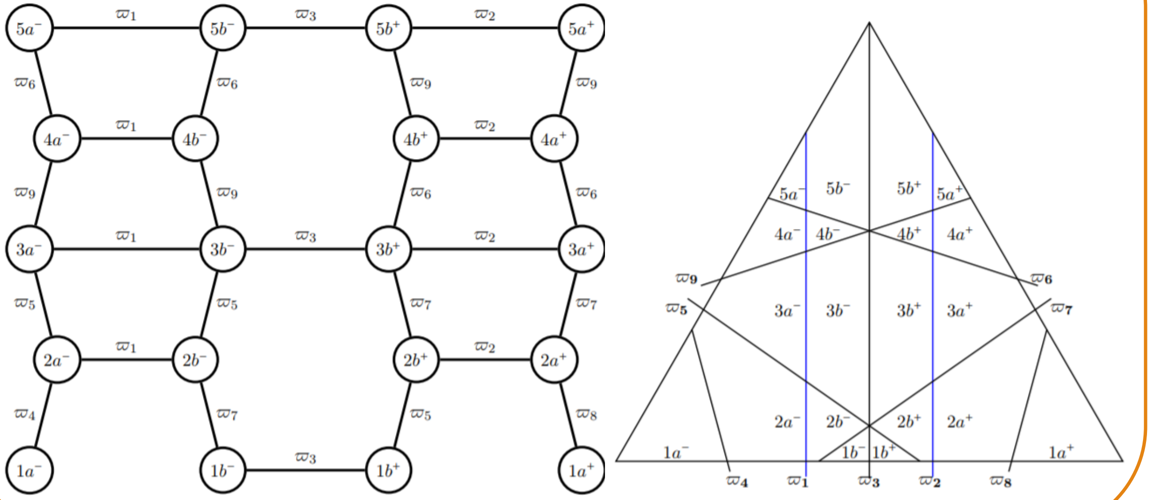
Algebra	Group	Kodaira Fiber	$h^{1,1}(Y_3)$	$h^{2,1}(Y_3)$	$\chi(Y_3)$
–	$\{e\}$	I_1	$11 - K^2$	$11 + 29K^2$	$-60K^2$
A_1	$SU(2)$	I_2^s, I_2^{ns}, III, IV^{ns}	$12 - K^2$	$12 + 29K^2 + 15KS + 3S^2$	$-60K^2 - 30KS - 6S^2$
A_2 G_2	$SU(3)$ G_2	I_3^s, IV^s I_0^{*ns}	$13 - K^2$	$13 + 29K^2 + 24KS + 6S^2$	$-60K^2 - 48KS - 12S^2$
A_3 B_3	$SU(4)$ $Spin(7)$	I_4^s I_0^{*ss}	$14 - K^2$	$14 + 29K^2 + 32KS + 10S^2$	$-60K^2 - 64KS - 20S^2$
D_4 F_4	$Spin(8)$ F_4	I_0^{*s} IV^{*ns}	$15 - K^2$	$15 + 29K^2 + 36KS + 12S^2$	$-60K^2 - 72KS - 24S^2$
A_4	$SU(5)$	I_5^s	$15 - K^2$	$15 + 29K^2 + 40KS + 15S^2$	$-60K^2 - 80KS - 30S^2$
D_5	$Spin(10)$	I_1^{*s}	$16 - K^2$	$16 + 29K^2 + 42KS + 16S^2$	$-60K^2 - 84KS - 32S^2$
E_6	E_6	IV^{*s}	$17 - K^2$	$17 + 29K^2 + 45KS + 18S^2$	$-60K^2 - 90KS - 36S^2$
E_7	E_7	III^*	$18 - K^2$	$18 + 29K^2 + 49KS + 21S^2$	$-60K^2 - 98KS - 42S^2$
E_8	E_8	II^*	$19 - K^2$	$19 + 29K^2 + 60KS + 30S^2$	$-60K^2 - 120KS - 60S^2$
A_1	$SO(3)$	I_2^{ns}	$12 - K^2$	$12 + 17K^2$	$-36K^2$
B_2	$SO(5)$	I_4^{ns}	$14 - K^2$	$14 + 9K^2$	$-20K^2$
A_3	$SO(6)$	I_4^s	$14 - K^2$	$14 + 5K^2$	$-12K^2$

Semi-simple Groups

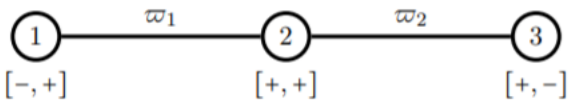
- The discriminant of the fibration contains at least two irreducible components Δ_1, Δ_2.
- *"Collisions of singularities"*

Models	Algebraic data	# Flops
$\mathrm{I}_2^{\text{ns}}+\mathrm{I}_4^{\text{ns}}$ MW = $\mathbb{Z}_2$	$F = y^2z - (x^3 + a_2x^2z + st^2xz^2)$ $\Delta = s^2t^4(a_2^2 - 4st^2)$ $G = (\mathrm{SU}(2)\times\mathrm{Sp}(4))/\mathbb{Z}_2$ $\mathbf{R} = \mathbf{(3,1)}\oplus\mathbf{(1,10)}\oplus\mathbf{(2,4)}\oplus\mathbf{(1,5)}$ $\chi = -4(9K^2 + 8K\cdot T + 3T^2)$	3
$\mathrm{I}_2^{\text{ns}}+\mathrm{I}_4^{\text{ns}}$ MW = $\{1\}$	$F = y^2z - (x^3 + a_2x^2z + \widetilde{a}_4st^2xz^2 + \widetilde{a}_6s^2t^4z^3)$ $\Delta = s^2t^4(4a_2^3\widetilde{a}_6 - a_2^2\widetilde{a}_4^2 - 18a_2\widetilde{a}_4\widetilde{a}_6st^2 + 4a_4^3st^2 + 27\widetilde{a}_6^2s^2t^4)$ $G = \mathrm{SU}(2)\times\mathrm{Sp}(4)$ $\mathbf{R} = \mathbf{(3,1)}\oplus\mathbf{(1,10)}\oplus\mathbf{(2,4)}\oplus\mathbf{(1,5)}\oplus\mathbf{(2,1)}\oplus\mathbf{(1,4)}$ $\chi = -2(30K^2 + 15K\cdot S + 30K\cdot T + 3S^2 + 8S\cdot T + 10T^2)$	3
$\mathrm{I}_2^{\text{ns}}+\mathrm{I}_4^{\text{s}}$ MW = $\mathbb{Z}_2$	$F = y^2z + a_1xyz - (x^3 + \widetilde{a}_2tx^2z + st^2xz^2)$ $\Delta = s^2t^4\left(a_1^4 + 8a_1^2\widetilde{a}_2t + 16\widetilde{a}_2^2t^2 - 64st^2\right)$ $G = (\mathrm{SU}(2)\times\mathrm{SU}(4))/\mathbb{Z}_2$ $\mathbf{R} = \mathbf{(3,1)}\oplus\mathbf{(1,15)}\oplus\mathbf{(2,4)}\oplus\mathbf{(2,\bar{4})}\oplus\mathbf{(1,6)}$ $\chi = -12\left(3K^2 + 3K\cdot T + T^2\right)$	12
$\mathrm{I}_2^{\text{ns}}+\mathrm{I}_4^{\text{s}}$ MW = $\{1\}$	$F = y^2z + a_1xyz - (x^3 + \widetilde{a}_2tx^2z + \widetilde{a}_4st^2xz^2 + \widetilde{a}_6s^2t^4z^3)$ $\Delta = s^2t^4\left(a_1^4 + 8a_1^2\widetilde{a}_2t + 16\widetilde{a}_2^2t^2 - 64st^2\right)$ $G = \mathrm{SU}(2)\times\mathrm{SU}(4)$ $\mathbf{R} = \mathbf{(3,1)}\oplus\mathbf{(1,15)}\oplus\mathbf{(2,4)}\oplus\mathbf{(2,\bar{4})}\oplus\mathbf{(1,6)}\oplus\mathbf{(2,1)}\oplus\mathbf{(1,4)}\oplus\mathbf{(1,\bar{4})}$ $\chi = -2\left(30K^2 + 15K\cdot S + 32K\cdot T + 3S^2 + 8S\cdot T + 10T^2\right)$	20

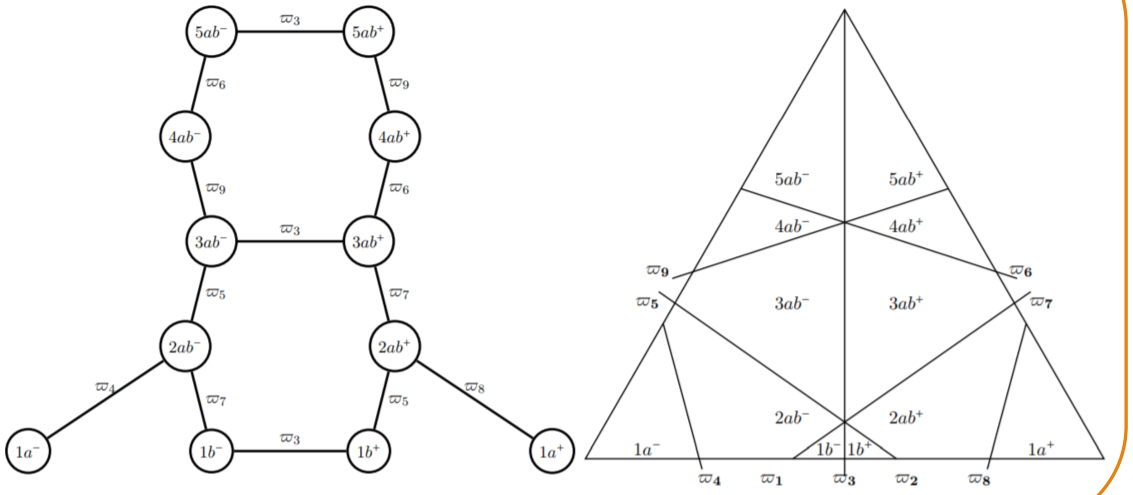
$G = \mathrm{SU}(2)\times\mathrm{SU}(4)$



$G = \mathrm{SU}(2)\times\mathrm{Sp}(4)$ $\quad$ $G = (\mathrm{SU}(2)\times\mathrm{Sp}(4))/\mathbb{Z}_2$



$G = (\mathrm{SU}(2)\times\mathrm{SU}(4))/\mathbb{Z}_2$



6d Anomaly Cancellation via Green-Schwarz

- Number of multiplets are given by: $n_T = 9 - K^2, \quad n_V = \dim G, \quad n_H^0 = h^{2,1}(Y) + 1.$
- Gravitational Anomalies are canceled when $n_H - n_V^{(6)} + 29n_T - 273 = 0.$
- For a semi-simple group with two simple components, $G = G_1 + G_2$, the remainder of the anomaly polynomial is given by

$$I_8 = \frac{K^2}{8}(\mathrm{tr}R^2)^2 + \frac{1}{6}(X_1^{(2)} + X_2^{(2)})\mathrm{tr}R^2 - \frac{2}{3}(X_1^{(4)} + X_2^{(4)}) + 4Y_{12}$$

where

$$\left\{ \begin{aligned} X_a^{(2)} &= \left(A_{a,\mathbf{adj}} - \sum_i n_{\mathbf{R_{i,a}}} A_{\mathbf{R_{i,a}}}\right) \mathrm{tr}_{\mathbf{F_a}} F_a^2, \\ X_a^{(4)} &= \left(B_{a,\mathbf{adj}} - \sum_i n_{\mathbf{R_{i,a}}} B_{\mathbf{R_{i,a}}}\right) \mathrm{tr}_{\mathbf{F_a}} F_a^4 + \left(C_{a,\mathbf{adj}} - \sum_i n_{\mathbf{R_{i,a}}} C_{\mathbf{R_{i,a}}}\right) (\mathrm{tr}_{\mathbf{F_a}} F_a^2)^2, \\ Y_{ab} &= \sum_{i,j} n_{\mathbf{R_{i,a}},\mathbf{R_{j,b}}} A_{R_{i,a}} A_{\mathbf{R_{j,b}}} \mathrm{tr}_{\mathbf{F_a}} F_a^2 \mathrm{tr}_{\mathbf{F_b}} F_b^2. \end{aligned} \right\}$$

- If the I_8 factors, then the anomalies are all canceled by Green-Schwarz mechanism.
- We check that all the anomalies are canceled!

Gravity without Supersymmetry

- Want to understand lower-dimensional gravitational theories without supersymmetry.
  - Hard to do with top-down approach
  - Holography (AdS/CFT) and Quantum Error Correction
- Want theories compatible with Reeh-Schlieder theorem.
  - Then this leads to infinite-dimensional Hilbert spaces!
- Reeh-Schlieder theorem:

  For any region A, by acting on the vacuum $|\Omega\rangle$ with operators located in that region we can produce a set of states which is dense in the full Hilbert space of the QFT.

Infinite-dimensional von Neumann Algebra

- Infinite-dimensional Hilbert space
  - Now we consider infinite-dimensional von Neumann algebra
- infinite-dimensional von Neumann algebra:

  An algebra of bounded operators that contains the identity operator, is closed under Hermitian conjugation, and is equal to its double commutant.

- Von Neumann algebra is naturally associated with causally complete spacetime regions.

open region of spacetime $\quad u \longrightarrow A(u) \quad$ an associated local operator algebra

Relative Entropy

- Infinite-dimensional Hilbert space: $S(\rho, \sigma) = \mathrm{Tr}\,(\rho \log \rho - \rho \log \sigma)$
- S(ρ,σ) does not increase upon performing a partial trace on ρ and σ.
  - The relative entropy may be intuitively thought of as a measure of distinguishability between two states.
- Infinite-dimensional case needs Tomita-Takesaki theory.

Relative Entropy and Tomita-Takesaki Theory

Let $|\Psi\rangle, |\Phi\rangle \in \mathcal{H}$ and M be a von Neumann algebra.

- The *relative Tomita operator* is the operator $S_{\Psi|\Phi}$ that acts as

$$S_{\Psi|\Phi} |x\rangle := |y\rangle$$

for any sequence $\{\mathcal{O}_n\} \in M$ such that the limits $|x\rangle = \lim_{n\to\infty} \mathcal{O}_n |\Psi\rangle$ and $|y\rangle = \lim_{n\to\infty} \mathcal{O}_n^\dagger |\Phi\rangle$ both exist.

- The *relative modular operator* is

$$\Delta_{\Psi|\Phi} := S^\dagger_{\Psi|\Phi} S_{\Psi|\Phi}.$$

- The *relative entropy* with respect to M of $|\Psi\rangle$ is

$$\mathcal{S}_{\Psi|\Phi}(M) = -\langle\Psi|\log \Delta_{\Psi|\Phi}|\Psi\rangle.$$

How do we utilize this?

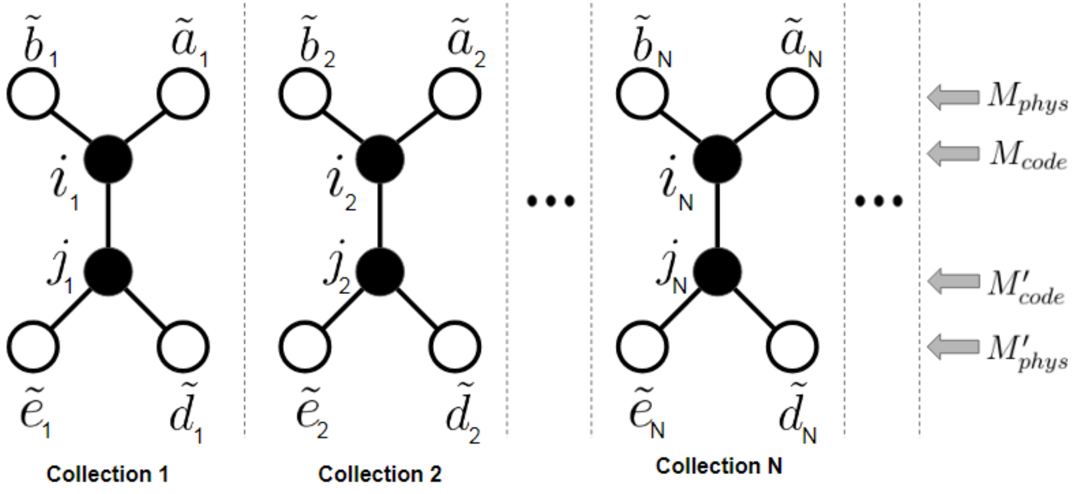
- We showed that for infinite-dimensional Hilbert spaces

  Entanglement Wedge Reconstruction

  $\leftrightarrow$ Equivalence of relative entropies between the boundary and the bulk

  1811.05482 (MJK, Kolchmeyer)

- Also, we have built an explicit quantum error correcting code that is of infinite-dimensional von Neumann algebra of type II_1.



Appearing on arXiv today!
(MJK, Kolchmeyer)

Conclusion

- We have extended understanding gravitational theories using two different views:
  - A top-down approach with F-theory and geometry of elliptic fibrations
  - An holographic understanding using infinite-dimensional von Neumann algebra
- Many more exciting further works to come!

Thank you for listening! ☺