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Characteristic numbers of crepant resolutions of Weierstrass models

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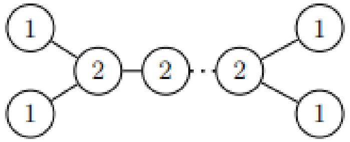
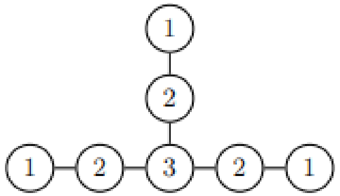
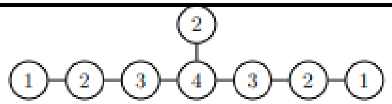
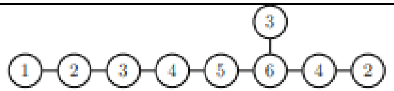
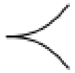
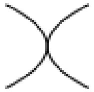
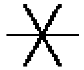
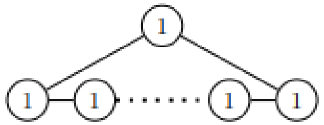
Elliptic Fibrations

- Weierstrass model: $y^2 z = x^3 + fxz^3 + gz^3$
  - Projective Bundle: $\pi : X_0 = P_B[\mathscr{O}_B \oplus \mathscr{L}^{\otimes 2} \oplus \mathscr{L}^{\otimes 3}] \to B$

    An elliptic fibration $\varphi : Y \to B$ cut out by the zero locus of a section of the line bundle $\mathscr{O}(3) \oplus \pi^* \mathscr{L}^{\otimes 6}$ in X_0, where $\mathscr{L}$ is a line bundle over a quasi-projective variety B.
  - Section: $z = x = 0$
  - Projective coordinates of the $\mathbb{P}^2$-fiber : $[z : x : y]$
  - Discriminant: $\Delta = 4f^3 + 27g^2$
  - j-invariant: $j = 1728 \dfrac{4f^3}{\Delta}$

Type	$v(c_4)$	$v(c_6)$	$v(\Delta)$	j	Monodromy	Fiber	Dual Graph
I_n^*	2	≥ 3	$n+6$	∞	$\begin{pmatrix} -1 & -n \\ 0 & -1 \end{pmatrix}$		$\tilde{D}_{n+4}$
	≥ 2	3	$n+6$				
IV^*	≥ 3	4	8	0	$\begin{pmatrix} -1 & -1 \\ 1 & 0 \end{pmatrix}$		$\tilde{E}_6$
III^*	3	≥ 5	9	1728	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$		$\tilde{E}_7$
II^*	≥ 4	5	10	0	$\begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix}$		$\tilde{E}_8$

Type	$v(c_4)$	$v(c_6)$	$v(\Delta)$	j	Monodromy	Fiber	Dual Graph
I_0	≥ 0	≥ 0	0	$\mathbb{C}$	I_2	Smooth	-
I_1	0	0	1	∞	$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$		$\tilde{A}_0$
II	≥ 1	1	2	0	$\begin{pmatrix} 1 & 1 \\ -1 & 0 \end{pmatrix}$		$\tilde{A}_0$
III	1	≥ 2	3	1728	$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$		$\tilde{A}_1$
IV	≥ 2	2	4	0	$\begin{pmatrix} 0 & 1 \\ -1 & -1 \end{pmatrix}$		$\tilde{A}_2$
I_n	0	0	$n > 1$	∞	$\begin{pmatrix} 1 & n \\ 0 & 1 \end{pmatrix}$		$\tilde{A}_{n-1}$

Kodaira Classification and Tate's Algorithm

G-model

Definition. Let G be a simple, simply-connected compact complex Lie group with Lie algebra $\mathfrak{g}$. A G-model is an elliptic fibration $\varphi : Y \to B$ with a discriminant locus containing an irreducible component S such that

1. the generic fiber over any other component of the discriminant is irreducible (that is, of Kodaira type I_1 or II),
2. the fiber over the generic point of S has a dual graph that becomes of the same type as the Dynkin diagram of the Langlands dual of $\mathfrak{g}$ after removing the node corresponding to the component touching the section of the elliptic fibration.

Invariance under crepant resolutions

- Need only a single crepant resolution to compute the Chern and Pontryagin numbers of a given G-model for the fourfolds.

$$\int_Y c_1(TY)^4, \int_Y c_1(TY)^2 c_2(TY), \int_Y c_1(TY) c_3(TY), \int_Y c_2^2(TY), \text{ and } \int_Y c_4(TY).$$

Theorem. *The Chern and Pontryagin numbers of an algebraic variety of complex dimension four are K-equivalence invariants.*

- Not generally true that Chern and Pontryagin numbers are invariant under crepant birational maps.
  - For fivefolds:

$$\boxed{\int c_1^5, \quad \int c_1^3 c_2, \quad \int c_1^2 c_3, \quad \int c_1 c_4, \quad \int c_5,} \quad \int c_1 c_2^2, \quad \int c_2 c_3$$

Only these are invariant

Strategy

- Find a crepant resolution for each G-model.
  - These crepant resolutions are obtained by sequence of blowups.
- Compute the Chern numbers and Pontryagin numbers using Pushforward formulas.
- Compute the remaining characteristic numbers:
  - all the holomorphic genera of the fourfolds,
  - the curvature invariant $X_8(Y)$,
  - the Hirzebruch signature $\sigma(Y)$,
  - the $\hat{A}$-genus.

Characteristic numbers (in terms of Chern numbers)

- Chern and Pontryagin numbers:

1. The Chern numbers

$$\int_Y c_1(TY)^4,\ \int_Y c_1(TY)^2 c_2(TY),\ \int_Y c_1(TY) c_3(TY),\ \int_Y c_2^2(TY),\ \text{and}\ \int_Y c_4(TY).$$

2. The Pontryagin numbers of a fourfold $\int_Y p_2(TY)$ and $\int_Y p_1^2(TY)$, where the Pontryagin classes are given by

$$p_1(TY) = c_1^2(TY) - 2c_2(TY),$$
$$p_2(TY) = c_2^2(TY) - 2c_1(TY)c_3(TY) + 2c_4(TY).$$

1. The holomorphic genera $\chi_p(Y) = \sum_{q=0}^{n}(-1)^q h^{p,q}(Y)$:

$$\chi_0(Y) = \int_Y \mathrm{Td}(TY) = \frac{1}{720}\int_Y (-c_4 + c_1c_3 + 3c_2^2 + 4c_1^2c_2 - c_1^4),$$
$$\chi_1(Y) = \frac{1}{180}\int_Y (-31c_4 - 14c_1c_3 + 3c_2^2 + 4c_1^2c_2 - c_1^4),$$
$$\chi_2(Y) = \frac{1}{120}\int_Y (79c_4 - 19c_1c_3 + 3c_2^2 + 4c_1^2c_2 - c_1^4).$$

2. The Hirzebruch signature of a fourfold,

$$\sigma(Y) = \frac{1}{45}\int_Y \left(7p_2(TY) - p_1^2(TY)\right) = \frac{1}{45}\int_Y (-c_1^4 + 4c_1^2c_2 + 3c_2^2 - 14c_1c_3 + 14c_4).$$

The signature is the degree of the Hirzebruch L-genus.

3. The Â-genus of a fourfold,

$$\int_Y \hat{A}_2(TY) = \frac{1}{5760}\int_Y \left(7p_1^2(TY) - 4p_2(TY)\right)$$
$$= \frac{1}{5760}\int_Y \left(7c_1^4 - 28c_1^2c_2 + 8c_1c_3 + 24c_2^2 - 8c_4\right).$$

By the Atiyah-Singer theorem, if the fourfold Y is a spin manifold, the degree of $\hat{A}_2$ gives the index of the Dirac operator on Y.

4. The curvature invarant 8-form,

$$X_8(Y) = \frac{1}{192}\int_Y \left(p_1^2(TY) - 4p_2(TY)\right).$$

G-models and their crepant resolutions

$\mathrm{I}_2^{\mathrm{s}}$	$\mathrm{SU}(2)$	:	$y^2 z + a_1 xyz + a_{3,1} syz = x^3 + a_{2,1} sx^2 z + a_{4,1} sxz^2 + a_{6,2} s^2 z^3,$
$\mathrm{I}_{2n}^{\mathrm{ns}}$	$\mathrm{USp}(2n)$	:	$y^2 z = x^3 + a_2 x^2 z + a_{4,n} s^n xz^2 + a_{6,2n} s^{2n} z^3,$
$\mathrm{I}_{2n+1}^{\mathrm{ns}}$	$\mathrm{USp}(2n)$	:	$y^2 z = x^3 + a_2 x^2 z + a_{4,n+1} s^{n+1} xz^2 + a_{6,2n+1} s^{2n+1} z^3,$
$\mathrm{I}_{2n}^{\mathrm{s}}$	$\mathrm{SU}(2n)$	:	$y^2 z + a_1 xyz = x^3 + a_{2,1} sx^2 z + a_{4,n} s^n xz^2 + a_{6,2n} s^{2n} z^3,$
$\mathrm{I}_{2n+1}^{\mathrm{s}}$	$\mathrm{SU}(2n+1)$	:	$y^2 z + a_1 xyz + a_{3,n} s^n yz^2 = x^3 + a_{2,1} sx^2 z + a_{4,n+1} s^{n+1} xz^2$
		:	$+ a_{6,2n+1} s^{2n+1} z^3,$
$\mathrm{I}_0^{*\mathrm{ss}}$	$\mathrm{Spin}(7)$	:	$y^2 z = x^3 + a_{2,1} sx^2 z + a_{4,2} s^2 xz^2 + a_{6,4} s^4 z^3,$
$\mathrm{I}_0^{*\mathrm{s}}$	$\mathrm{Spin}(8)$	:	$y^2 z = (x - x_1 sz)(x - x_2 sz)(x - x_3 sz) + s^2 r x^2 z + s^3 q xz^2 + s^4 t z^3,$
III	$\mathrm{SU}(2)$	:	$y^2 z = x^3 + s a_{4,1} xz^2 + s^2 a_{6,2} z^3,$
$\mathrm{IV}^{\mathrm{ns}}$	$\mathrm{SU}(2)$	:	$y^2 z = x^3 + s^2 a_{4,2} xz^2 + s^2 a_{6,2} z^3,$
IV^{s}	$\mathrm{SU}(3)$	:	$y^2 z + a_{3,1} syz^2 = x^3 + s^2 a_{4,2} xz^2 + s^3 a_{6,3} z^3,$
$\mathrm{I}_0^{*\mathrm{ns}}$	G_2	:	$y^2 z = x^3 + s^2 a_{4,2} xz^2 + s^3 a_{6,3} z^3,$
$\mathrm{IV}^{*\mathrm{ns}}$	F_4	:	$y^2 z = x^3 + s^3 a_{4,3} xz^2 + s^4 a_{6,4} z^3,$
$\mathrm{IV}^{*\mathrm{s}}$	E_6	:	$y^2 z + a_{3,2} s^2 yz^2 = x^3 + s^3 a_{4,3} xz^2 + s^5 a_{6,5} z^3,$
III^*	E_7	:	$y^2 z = x^3 + s^3 a_{4,3} xz + s^5 a_{6,5} z^3,$
II^*	E_8	:	$y^2 z = x^3 + s^4 a_{4,4} xz^2 + s^5 a_{6,5} z^3.$

Group	Fiber Type	Crepant Resolution
$\mathrm{SU}(2)$	$\mathrm{I}_2^{\mathrm{s}}, \mathrm{I}_2^{\mathrm{ns}}$ $\mathrm{I}_3^{\mathrm{ns}}, \mathrm{III}$ $\mathrm{IV}^{\mathrm{ns}}$	$X_0 \xleftarrow{(x,y,s\vert e_1)} X_1$
$\mathrm{SU}(3)$ $\mathrm{USp}(4)$ G_2	$\mathrm{I}_3^{\mathrm{s}}, \mathrm{IV}^{\mathrm{s}}$ $\mathrm{I}_4^{\mathrm{ns}}$ $\mathrm{I}_0^{*\mathrm{ns}}$	$X_0 \xleftarrow{(x,y,s\vert e_1)} X_1 \xleftarrow{(y,e_1\vert e_2)} X_2$
$\mathrm{SU}(4)$ $\mathrm{Spin}(7)$	$\mathrm{I}_4^{\mathrm{s}}$ $\mathrm{I}_0^{*\mathrm{ss}}$	$X_0 \xleftarrow{(x,y,s\vert e_1)} X_1 \xleftarrow{(y,e_1\vert e_2)} X_2 \xleftarrow{(x,e_2\vert e_3)} X_3$
$\mathrm{Spin}(8)$	$\mathrm{I}_0^{*\mathrm{s}}$	$X_0 \xleftarrow{(x,y,s\vert e_1)} X_1 \xleftarrow{(y,e_1\vert e_2)} X_2 \xleftarrow{(x - x_i sz, e_2\vert e_3)} X_3 \xleftarrow{(x - x_j sz, e_2\vert e_4)} X_4$
F_4	$\mathrm{IV}^{*\mathrm{ns}}$	$X_0 \xleftarrow{(x,y,s\vert e_1)} X_1 \xleftarrow{(y,e_1\vert e_2)} X_2 \xleftarrow{(x,e_2\vert e_3)} X_3 \xleftarrow{(e_3,e_2\vert e_4)} X_4$
$\mathrm{SU}(5)$	$\mathrm{I}_5^{\mathrm{s}}$	$X_0 \xleftarrow{(x,y,s\vert e_1)} X_1 \xleftarrow{(x,y,e_1\vert e_2)} X_2 \xleftarrow{(y,e_1\vert e_3)} X_3 \xleftarrow{(y,e_2\vert e_4)} X_4$
$\mathrm{Spin}(10)$	$\mathrm{I}_1^{*\mathrm{s}}$	$X_0 \xleftarrow{(x,y,s\vert e_1)} X_1 \xleftarrow{(y,e_1\vert e_2)} X_2 \xleftarrow{(x,e_2\vert e_3)} X_3 \xleftarrow{(y,e_3\vert e_4)} X_4 \xleftarrow{(e_2,e_3\vert e_5)} X_5$
$\mathrm{SU}(6)$	$\mathrm{I}_6^{\mathrm{s}}$	$X_0 \xleftarrow{(x,y,s\vert e_1)} X_1 \xleftarrow{(y,e_1\vert e_2)} X_2 \xleftarrow{(x,e_2\vert e_3)} X_3 \xleftarrow{(y,e_3\vert e_4)} X_4 \xleftarrow{(x,e_4\vert e_5)} X_5$
$\mathrm{SU}(7)$	$\mathrm{I}_7^{\mathrm{s}}$	$X_0 \xleftarrow{(x,y,s\vert e_1)} X_1 \xleftarrow{(y,e_1\vert e_2)} X_2 \xleftarrow{(x,e_2\vert e_3)} X_3 \xleftarrow{(y,e_3\vert e_4)} X_4$ $X_4 \xleftarrow{(x,e_4\vert e_5)} X_5$ (↑) $X_6 \xrightarrow{(y,e_5\vert e_6)} X_5$
E_6	$\mathrm{IV}^{*\mathrm{s}}$	$X_0 \xleftarrow{(x,y,s\vert e_1)} X_1 \xleftarrow{(y,e_1\vert e_2)} X_2 \xleftarrow{(x,e_2\vert e_3)} X_3 \xleftarrow{(e_2,e_3\vert e_4)} X_4$ $X_4 \xleftarrow{(y,e_3\vert e_5)} X_5$ (↑) $X_6 \xrightarrow{(y,e_4\vert e_6)} X_5$
E_7	III^*	$X_0 \xleftarrow{(x,y,s\vert e_1)} X_1 \xleftarrow{(y,e_1\vert e_2)} X_2 \xleftarrow{(x,e_2\vert e_3)} X_3 \xleftarrow{(y,e_3\vert e_4)} X_4$ $X_4 \xleftarrow{(e_2,e_3\vert e_5)} X_5$ (↑) $X_7 \xrightarrow{(e_4,e_5\vert e_7)} X_6 \xrightarrow{(e_2,e_4\vert e_6)} X_5$
E_8	II^*	$X_0 \xleftarrow{(x,y,s\vert e_1)} X_1 \xleftarrow{(y,e_1\vert e_2)} X_2 \xleftarrow{(x,e_2\vert e_3)} X_3 \xleftarrow{(y,e_3\vert e_4)} X_4$ $X_4 \xleftarrow{(e_2,e_3\vert e_5)} X_5$ (↑) $X_8 \xrightarrow{(e_4,e_7\vert e_8)} X_7 \xrightarrow{(e_2,e_4,e_6\vert e_7)} X_6 \xrightarrow{(e_4,e_5\vert e_6)} X_5$

Pushforward formula-1

➢ The following theorem gives the total Chern class after a blowup along a local complete intersection.

Theorem (Aluffi). *Let $Z \subset X$ be the complete intersection of d nonsingular hypersurfaces $Z_1, \ldots, Z_d$ meeting transversally in X. Let $f : \widetilde{X} \longrightarrow X$ be the blowup of X centered at Z. We denote the exceptional divisor of f by E. The total Chern class of $\widetilde{X}$ is then:*

$$c(T\widetilde{X}) = (1+E)\left(\prod_{i=1}^{d} \frac{1+f^*Z_i - E}{1+f^*Z_i}\right) f^*c(TX).$$

Pushforward formula-2

- The following theorem provides a user-friendly method to compute invariants of the blown-up space in terms of the original space.

Theorem (Esole–Jefferson–Kang). *Let the nonsingular variety $Z \subset X$ be a complete intersection of d nonsingular hypersurfaces $Z_1, \ldots, Z_d$ meeting transversally in X. Let E be the class of the exceptional divisor of the blowup $f : \widetilde{X} \longrightarrow X$ centered at Z. Let $\widetilde{Q}(t) = \sum_a f^* Q_a t^a$ be a formal power series with $Q_a \in A_*(X)$. We define the associated formal power series $Q(t) = \sum_a Q_a t^a$, whose coefficients pullback to the coefficients of $\widetilde{Q}(t)$. Then the pushforward $f_* \widetilde{Q}(E)$ is*

$$f_* \widetilde{Q}(E) = \sum_{\ell=1}^{d} Q(Z_\ell) M_\ell, \quad \text{where} \quad M_\ell = \prod_{\substack{m=1 \\ m \neq \ell}}^{d} \frac{Z_m}{Z_m - Z_\ell}.$$

Pushforward formula-3

- This theorem gives a simple method to pushforward analytic expressions in the Chow ring of the projective bundle X_0 to the Chow ring of its base.
- It is a direct consequence of functorial properties of the Segre class.

Theorem (Esole–Jefferson–Kang). *Let $\mathscr{L}$ be a line bundle over a variety B and $\pi : X_0 = \mathbb{P}[\mathscr{O}_B \oplus \mathscr{L}^{\otimes 2} \oplus \mathscr{L}^{\otimes 3}] \longrightarrow B$ a projective bundle over B. Let $\widetilde{Q}(t) = \sum_a \pi^* Q_a t^a$ be a formal power series in t such that $Q_a \in A_*(B)$. Define the auxiliary power series $Q(t) = \sum_a Q_a t^a$. Then*

$$\pi_* \widetilde{Q}(H) = -2 \left. \frac{Q(H)}{H^2} \right|_{H=-2L} + 3 \left. \frac{Q(H)}{H^2} \right|_{H=-3L} + \frac{Q(0)}{6L^2},$$

where $L = c_1(\mathscr{L})$ and $H = c_1(\mathscr{O}_{X_0}(1))$ is the first Chern class of the dual of the tautological line bundle of $\pi : X_0 = \mathbb{P}(\mathscr{O}_B \oplus \mathscr{L}^{\otimes 2} \oplus \mathscr{L}^{\otimes 3}) \to B$.

Chern and Pontryagin numbers of G-models

Theorem. *For a G-model, defined by a crepant resolution of a singular Weierstrass model, we have*

$$\int_Y c_2^2(TY) = 24\int_B L(c_2 - c_1 L + 6L^2) + \mu_G,$$
$$\int_Y p_2(TY) = 24\int_B L(2c_2 - c_1^2 + 36L^2) + 7\mu_G,$$
$$\int_Y p_1^2(TY) = 48\int_B L(2c_2 - c_1^2 + 11L^2) + 4\mu_G,$$

where $\mu_G = \int_B S(\alpha_0 L^2 + \alpha_1 LS + \alpha_2 S^2)$ *is the contribution from the singularities induced by the Kodaira type over* S.

Contributions from singularities

Algebra	$\mu_G = \int_Y c_2^2(TY) - \int_{Y_0} c_2^2(TY_0)$
A_1	$-2\int_B S(7L-S)^2$
$A_2,\ C_2,\ G_2$	$-8\int_B S\left(19L^2-8LS+S^2\right)$
$A_3,\ B_3$	$-4\int_B S\left(50L^2-28LS+5S^2\right)$
$D_4,\ F_4$	$-8\int_B S\left(27L^2-16LS+3S^2\right)$
A_4	$-5\int_B S\left(50L^2-35LS+8S^2\right)$
D_5	$-4\int_B S\left(63L^2-44LS+10S^2\right)$
A_5	$-\int_B S\left(298L^2-251LS+70\right)$
A_6	$-2\int_B S\left(174L^2-171LS+56S^2\right)$
E_6	$-3\int_B S\left(86L^2-61LS+14S^2\right)$
E_7	$-\int_B \left(135L^2-100LS+24S^2\right)$
E_8	$-\int_B \left(8L^2-7LS+2S^2\right)$

Holomorphic genera of the fourfolds

- The divisor S is the one supporting the reducible Kodaira fiber corresponding to the type of the Lie algebra $\mathfrak{g}$.
- $L = c_1(\mathscr{L})$ and c_i denotes the ith Chern class of the base of the fibration.
- The holomorphic Euler characteristic $\chi_0(Y)$ is equal to $\chi_0(W, \mathscr{O}_W)$ where W is the Weierstrass divisor defined by $\mathscr{L}$ in the base.

Group	χ_0	χ_1	χ_2
SU(2)	$\frac{1}{12}L\,(c_1^2 - 3c_1L + c_2 + 2L^2)$	$-\frac{1}{3}L\,(2c_1^2 - 54c_1L + 5c_2 + 232L^2)$ $+\frac{1}{2}S^2(3c_1 - 31L) - \frac{1}{2}LS(15c_1 - 113L) + S^3$	$-\frac{1}{2}L\,(3c_1^2 + 71c_1L - 17c_2 - 554L^2)$ $+S^2(59L - 3c_1) + LS(15c_1 - 211L) - 4S^3$
SU(3) USp(4) G_2	$\frac{1}{12}L\,(c_1^2 - 3c_1L + c_2 + 2L^2)$	$-\frac{1}{3}L\,(2c_1^2 - 54c_1L + 5c_2 + 232L^2)$ $+S^2(3c_1 - 35L) - 4LS(3c_1 - 22L) + 4S^3$	$-\frac{1}{2}L\,(3c_1^2 + 71c_1L - 17c_2 - 554L^2)$ $-2S^2(3c_1 - 67L) + 8LS(3c_1 - 41L) - 16S^3$
SU(4) Spin(7)	$\frac{1}{12}L\,(c_1^2 - 3c_1L + c_2 + 2L^2)$	$-\frac{1}{3}L\,(2c_1^2 - 54c_1L + 5c_2 + 232L^2)$ $+S^2(5c_1 - 61L) - 4LS(4c_1 - 29L) + 10S^3$	$-\frac{1}{2}L\,(3c_1^2 + 71c_1L - 17c_2 - 554L^2)$ $-2S^2(5c_1 - 117L) + 16LS(2c_1 - 27L) - 40S^3$
Spin(8) F_4	$\frac{1}{12}L\,(c_1^2 - 3c_1L + c_2 + 2L^2)$	$-\frac{1}{3}L\,(2c_1^2 - 54c_1L + 5c_2 + 232L^2)$ $+2S^2(3c_1 - 35L) - 18LS(c_1 - 7L) + 12S^3$	$-\frac{1}{2}L\,(3c_1^2 + 71c_1L - 17c_2 - 554L^2)$ $-4S^2(3c_1 - 67L) + 36LS(c_1 - 13L) - 48S^3$
SU(5)	$\frac{1}{12}L\,(c_1^2 - 3c_1L + c_2 + 2L^2)$	$-\frac{1}{3}L\,(2c_1^2 - 54c_1L + 5c_2 + 232L^2)$ $+\frac{5}{2}S^2(3c_1 - 38L) - 5LS(4c_1 - 29L) + 20S^3$	$-\frac{1}{2}L\,(3c_1^2 + 71c_1L - 17c_2 - 554L^2)$ $-5S^2(3c_1 - 73L) + 20LS(2c_1 - 27L) - 80S^3$
Spin(10)	$\frac{1}{12}L\,(c_1^2 - 3c_1L + c_2 + 2L^2)$	$-\frac{1}{3}L\,(2c_1^2 - 54c_1L + 5c_2 + 232L^2)$ $+8S^2(c_1 - 12L) - 21LS(c_1 - 7L) + 20S^3$	$-\frac{1}{3}L\,(2c_1^2 - 54c_1L + 5c_2 + 232L^2)$ $+8S^2(c_1 - 12L) - 21LS(c_1 - 7L) + 20S^3$
SU(6)	$\frac{1}{12}L\,(c_1^2 - 3c_1L + c_2 + 2L^2)$	$-\frac{1}{3}L\,(2c_1^2 - 54c_1L + 5c_2 + 232L^2)$ $+\frac{1}{2}S^2(21c_1 - 272L) - LS(24c_1 - 173L) + 35S^3$	$-\frac{1}{2}L\,(3c_1^2 + 71c_1L - 17c_2 - 554L^2)$ $+S^2(523L - 21c_1) + 4LS(12c_1 - 161L) - 140S^3$
SU(7)	$\frac{1}{12}L\,(c_1^2 - 3c_1L + c_2 + 2L^2)$	$-\frac{1}{3}L\,(2c_1^2 - 54c_1L + 5c_2 + 232L^2)$ $(+S^2)(14c_1 - 185L) - 2LS(14c_1 - 101L) + 56S^3$	$-\frac{1}{2}L\,(3c_1^2 + 71c_1L - 17c_2 - 554L^2)$ $-4S^2(7c_1 - 178L) + 8LS(7c_1 - 94L) - 224S^3$
E_6	$\frac{1}{12}L\,(c_1^2 - 3c_1L + c_2 + 2L^2)$	$-\frac{1}{3}L\,(2c_1^2 - 54c_1L + 5c_2 + 232L^2)$ $+\frac{3}{2}S^2(6c_1 - 67L) - \frac{3}{2}LS(15c_1 - 101L) + 21S^3$	$-\frac{1}{2}L\,(3c_1^2 + 71c_1L - 17c_2 - 554L^2)$ $-6S^2(3c_1 - 64L) + 3LS(15c_1 - 187L) - 84S^3$
E_7	$\frac{1}{12}L\,(c_1^2 - 3c_1L + c_2 + 2L^2)$	$-\frac{1}{3}L\,(2c_1^2 - 54c_1L + 5c_2 + 232L^2)$ $+\frac{1}{2}S^2(21c_1 - 221L) - \frac{1}{2}LS(49c_1 - 319L) + 24S^3$	$-\frac{1}{2}L\,(3c_1^2 + 71c_1L - 17c_2 - 554L^2)$ $+S^2(421L - 21c_1) + LS(49c_1 - 589L) - 96S^3$
E_8	$\frac{1}{12}L\,(c_1^2 - 3c_1L + c_2 + 2L^2)$	$-\frac{1}{3}L\,(2c_1^2 - 54c_1L + 5c_2 + 232L^2)$ $+5S^2(3c_1 - 31L) + 10LS(19L - 3c_1) + 40S^3$	$\frac{1}{2}L\,(-3c_1^2 - 71c_1L + 17c_2 + 554L^2)$ $+10S^2(59L - 3c_1) + 20LS(3c_1 - 35L) - 160S^3$

More characteristic numbers of G-models

Theorem. *For a G-model, we have the following characteristic numbers of the fourfolds:*

$$192\ X_8(Y) = \int_Y \left(p_1^2(TY) - 4p_2(TY)\right) = 48\int_B L\left(c_1^2 - 2c_2 - 61L^2\right) - 24\mu_G,$$

$$45\ \sigma(Y) = \int_Y \left(7p_2(TY) - p_1^2(TY)\right) = 120\int_B L\left(2c_2 - c_1^2 + 46L^2\right) + 45\mu_G,$$

$$5760\ \int_Y \hat{A}_2(TY) = \int_Y \left(7p_1^2(TY) - 4p_2(TY)\right) = 240\int_B L\left(2c_2 - c_1^2 + L^2\right).$$

Calabi-Yau fourfolds

- The only nonzero Chern numbers are

$$\int_Y c_2^2(TY) = 480 + \frac{1}{3}\chi(Y), \quad \int_Y c_4(TY) = \chi(Y).$$

- All Chern and Pontryagin numbers are topological invariants as they are functions of the Euler characteristic of Calabi-Yau fourfolds.

- Then, all the characteristic numbers are functions of the Euler characteristic of Calabi-Yau fourfolds:

$$\int_Y c_2^2(TY) = 480 + \frac{1}{3}\chi(Y), \quad \sigma = 32 + \frac{1}{3}\chi(Y), \quad \chi_0 = 2, \quad \chi_1 = 8 - \frac{1}{6}\chi(Y),$$
$$\chi_2 = 12 + \frac{2}{3}\chi(Y), \quad X_8 = -\frac{1}{24}\chi(Y), \quad \frac{1}{5760}\int_Y \hat{A}_2 = 2.$$

Thank you for listening! ☺