The Compton scattering of an x-ray is incoherent because there are degrees of freedom in each scattering event associated with the atomic electron. Compton scattering provides a background intensity in x-ray diffraction patterns that can be understood as follows. The outer electrons of an atom are the ones that can participate in Compton scattering because they can become unbound from the atom and carry momentum when they acquire the energy of $h\Delta\nu$. Compton scattering by outer electrons is more likely at higher diffraction angles 2θ, where $h\Delta\nu$ is as large as 125 eV for Cu $K\alpha$ radiation, for example. The Compton background therefore rises with 2θ angle. The core electrons of heavier atoms do not participate in Compton scattering, since they are bound too tightly. The relative amount of Compton scattering versus coherent scattering therefore decreases with the atomic number of the element. It turns out that the total inelastic Compton scattering intensity plus the total elastic intensity are exactly equal to the Thompson scattering.

3.2.3 X-Ray Mass Attenuation Coefficients

As an x-ray beam passes through a material, the energy of each x-ray remains constant, but there is a decrease in number of the incident x-rays. At the depth x, the increment of thickness of a material, $\mathrm{d}x$, scatters a number of x-rays, $\mathrm{d}I$, removing them from the beam. The number of lost x-rays, $-\mathrm{d}I(x)$, equals the product of 1) the increment of thickness, $\mathrm{d}x$, 2) the number of x-rays present at x, $I(x)$, and 3) a material coefficient, μ:

$$-\mathrm{d}I(x) = \mu I(x)\mathrm{d}x \ , \tag{3.36}$$

$$\frac{\mathrm{d}I(x)}{\mathrm{d}x} = -\mu I(x) \ , \tag{3.37}$$

$$I(x) = I_0 \mathrm{e}^{-\mu x} \ . \tag{3.38}$$

The product in the exponent, μx, must be dimensionless, so μ has dimensions of [cm^{-1}]. When μx is small, it equals the fraction of x-rays removed from the incident beam. From Fig. 3.1 we know that this fraction also equals $N\sigma/A$, so:

$$\mu = \frac{N\sigma}{Ax} = \frac{N}{V}\sigma \ , \tag{3.39}$$

where N/V has units [atoms cm^{-3}] and σ is the scattering cross-section with units [cm^2]. Since density varies with the type of material, tabulations such as the one in Appendix A.2 provide "mass attenuation coefficients," which are ratios μ/ρ. Here the density, ρ, has units [$\mathrm{g\,cm}^{-3}$], so the coefficients μ/ρ have units [cm^{-1}]/[$\mathrm{g\,cm}^{-3}$]=[cm^2 g^{-1}]. Exponents in 3.38 are products $(\mu/\rho) \times \rho \times x$, and are, of course, dimensionless.

As a typical application of mass attenuation coefficients tabulated in Appendix A.2, consider the characteristic depth of penetration for Cu $K\alpha$ x-rays in a sample of iron metal. This is obtained readily: the mass attenuation coefficient is 302 $\mathrm{g}^{-1}\,\mathrm{cm}^2$, the density of iron is 7.86 $\mathrm{g\,cm}^{-3}$, and the inverse of

the product of these numbers gives 4.2 μm. For comparison, the table also shows that higher energy Mo $K\alpha$ x-rays are more penetrating in iron, having an e^{-1} reduction in intensity ($e^{-1} = 0.368$) over a distance of 34 μm.

It is straightforward to calculate the composite mass attenuation coefficient for a compound or an alloy. (We obtain a different expression from (1.67), however, which involved multiple phases.) In all absorption problems, the point to remember is that the net x-ray scattering depends on the number and types of atoms in the path of the beam. The composite mass attenuation coefficient is obtained from the mass attenuation coefficients, μ_i, for the different elements, i, weighted by their atomic fractions in the material, f_i:

$$< \mu >= \sum_i f_i \, \mu_i \; . \tag{3.40}$$

For use with tabulated values of μ/ρ, however, we must use mass fractions. For example, consider the attenuation of Cu $K\alpha$ radiation in an Fe-25at.%Al alloy, which has a density of $6.8\,\mathrm{g\,cm^{-3}}$. We attribute 13.9% of the density to Al and 86.1% to the Fe because the alloy composition is Fe-13.9 wt.% Al. For Cu $K\alpha$ radiation the product, $< \mu\rho >_{\mathrm{FeAl}}$, is:

$$< \mu\rho >_{\mathrm{FeAl}} = \left[0.139 \cdot 49.6 + 0.861 \cdot 302 \right] 6.8 = 1815 \ \mathrm{cm^{-1}} \; . \tag{3.41}$$

This gives a characteristic length of 5.5 μm. Interestingly, if we assume that the scattering is due entirely to iron, we obtain a characteristic length of 5.7 μm. In this example the mass attenuation is dominated by the iron in the material, primarily because iron is the stronger x-ray attenuator (and secondarily because iron is the majority species). Figure 3.4 is an x-ray penetration image of an important work of art, "Blue Boy," by Thomas Gainsborough. Many minerals are used in paint pigments, but in Gainsborough's day the mineral lead carbonate was used for the color white. The lead dominates the x-ray absorption, and in this (negative) image the light regions correspond to a high lead density.[7]

The material coefficient, μ, originates with both inelastic and elastic scattering. For x-rays with energies from 1 to 20 keV, however, the mass attenuation coefficient is dominated by photoelectric absorption, where an incident x-ray loses energy by exciting an electron out of the atom. Photoelectric absorption requires the energy of the incident x-ray to be greater than the binding energy of an atomic electron. The mass absorption coefficients are larger for elements where the x-ray energy exceeds a binding energy of an atomic electron. For Cu $K\alpha$ x-rays, for example, this causes a 7-fold increase in mass absorption coefficient for Co over that of Ni. The energy of a Cu $K\alpha$ x-ray is 8.05 keV, whereas the energy required for exciting a K-electron from Co is 7.71 keV, and from Ni it is 8.33 keV.

[7] Notice the dog in the lower right, which Gainsborough evidently decided was inappropriate for the portrait. The top of the x-ray image also shows the collar of another person, indicating the canvas itself was used for a previous portrait.

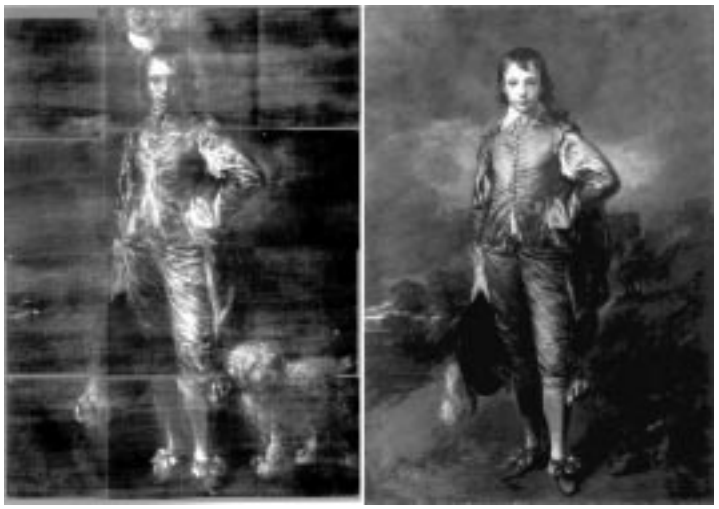

Fig. 3.4. Left: Negative image of x-ray penetration through the canvas "Blue Boy," by Thomas Gainsborough. Right: The portrait surface photographed with reflected light. After [3.1].

3.3 Coherent Elastic Scattering

3.3.1 ‡ Born Approximation for Electrons

Almost without a second thought, we treat electron scattering as a wave phenomenon with the electron wavefunction satisfying the Schrödinger wave equation. An electron diffraction pattern, with its series of spots or rings as in Fig. 1.8, is certainly evidence of wave behavior. The interpretation of the electron wavefunction is different from that of a simple wave, however. Suppose we were to turn on an electron beam and watch the formation of the diffraction pattern of Fig. 1.8, using a detector capable of recording impacts of individual electrons. When the electron beam is turned on, a series of bright flashes are observed at various points on the detector screen. Each individual event occurs at a particular point on the detector, and does not appear as a continuous ring. With time, an obvious bias appears, where the points of detection are most frequently at the positions of the rings and spots of the diffraction pattern. This behavior motivates the interpretation of the electron wavefunction in terms of probabilities – specifically, the electron probability is the electron wavefunction times its complex conjugate (which makes a real number). Usually this probabilistic interpretation can be ignored when we consider a diffraction pattern from many electrons, and we can consider electron diffraction as the diffraction of any other type of wave. When individual electron events are considered, however, we may have to recall the probabilistic interpretation of the electron wavefunction because individual electron detections look like particles rather than waves.

Another point to remember is that wave behavior is a characteristic of an *individual* electron. When considering a diffraction pattern involving multiple electrons, we do not add the amplitudes of multiple wavefunctions. At the viewing screen, we add the intensities of individual electrons. The interactions between different high-energy electrons are not coherent.